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The effect of a strong longtitudinal magnetic field on the flow of mercury in a circular tube

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An experimental study of the effects of large longitudinal magnetic fields on the pipe flow of mercury has been made. The experiments were designed to provide large magnetic interaction parameters and thereby to have an important effect upon transition from laminar to turbulent flow and upon the structure of any turbulence. The friction factor and the transition Reynolds number were measured as functions of the magnetic field strength. The friction factor was obtained from accurate measurement of the pressure gradient along the tube. The transition Reynolds number was determined from measurements of the intermittency factor obtained using a hot-wire anemometer.

The results indicate that all relevant stability theories vastly over-estimate the stabilizing effect of the field. The results also indicate that viscosity plays a critical role in delaying transition and in reducing the friction factor of fully developed flows. This, in turn, leads to the important conclusion that the magnetic field has an important effect on the generation of new turbulence, as well as upon the damping of already generated turbulence.

1. Introduction

The interaction of laminar and turbulent flows of electrically conducting fluids with magnetic fields, magnetohydrodynamics (MHD), has been the subject of a great deal of theoretical study and, to a lesser extent, experimental investigation. Experimental work has lagged because of problems involved in working with electrically conducting fluids, both gases and liquids, and in obtaining magnetic effects which are large enough to change significantly the character of the flow.

The most extensively studied area of MHD has been that of channel and pipe flows, mainly because of the practical importance of these flows, and because of their close connexion with ordinary pipe flows. The greatest portion of this work has been concerned with flows having transverse magnetic fields. This is the configuration found in MHD generators, accelerators, pumps and flow meters. The most notable experiments with this geometry were performed using liquid metals by Hartmann & Lazarus (1937) and Murgatroyd (1953). The first analysis of the transverse field case was performed by Hartmann (1937). This analysis

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was primarily concerned with laminar flow. The question of the stability of transverse field flows was considered by Lock (1955), while Harris (1960) used the available turbulent data to predict velocity profiles and current densities in channel flows.

The most important characteristic of the transverse field MHD channel flow is the coupling between the mean velocity profile and the magnetic field. The magnetic field strongly influences the mean velocity profile whether the flow is laminar or turbulent, and it is this coupling which governs the behaviour of the flow. Therefore, in the fully developed turbulent flow, the magnetic effects are due mainly to the coupling rather than to the damping of turbulent eddies. It is difficult to examine the effect of the field upon the turbulence in such a flow because the damping is a secondary characteristic.

A more interesting flow from the point of view of short-range MHD effects is that for which the mean flow and magnetic field are aligned. Such a flow does not exhibit the coupling between the field and mean flow which is of such great importance when the field is transverse. A longitudinal field has no effect on the fully developed laminar pipe flow because the field and the flow are parallel. Therefore, the field should influence the flow only when the flow becomes turbulent, or when small disturbances exist in an otherwise laminar flow. In turbulent flow, the magnetic field opposes transverse velocities and thus tends to damp the turbulence. Since the longitudinal field cannot exert longitudinal forces on the flow, it does not affect the mean velocity profile directly. However, it does affect the mean velocity through modification of the turbulent structure of the flow. Consequently, flows of this type reveal directly the influence of the fields on the turbulent structure.

Compared to the extensive body of analytical and experimental work carried out for transverse field channel flows, little work has been done for flows with longitudinal fields. There have been two previous experimental studies of such flows, both experiments using round tubes rather than rectangular channels. The first experiment, performed by Bader & Carlson (1958), reported little or no change in the character of the flow; in particular, the field did not appear to affect the laminar to turbulent transition point. The second experiment, in which stronger effects were produced, was performed by Globe (1961). Globe reported a change in the Reynolds number for transition to turbulent flow, turbulent flow being delayed to higher Reynolds numbers as the field became stronger. He also noted a reduction in the turbulent friction factor, although this was not of major interest in his experiment.

Little analytical work has been done which applies directly to pipe type flow with an aligned field. Stability analyses yielding similar results have been performed for flows between parallel planes by Rossow (1958), Stuart (1954), Hunt (1966), Michael (1953) and Hains (1965), among others. Ordinary hydrodynamic flow stability analyses for parallel flow indicate complete stability, however, and consequently shed no light upon the transition of pipe flow. It should not be anticipated, therefore, that the corresponding MHD analyses should apply directly to pipe flow. It is unlikely that this situation can be improved before the ordinary pipe flow stability analysis is successfully performed.

No thorough analysis of fully developed turbulent flow with an axial magnetic field has been undertaken. Several studies have been made of flows with weak magnetic effects. These analyses have been limited to extension of ordinary flow theories, such as the mixing-length theory. The most interesting results would be for the strong magnetic interaction case, for which no analytical results are available. The present study was undertaken to provide the fundamental data upon which further analyses may be firmly based.

2. Parametric study

The two previous longitudinal field pipe flow experiments served to guide the design of the present experiment. The results of these experiments, expressed in terms of appropriate dimensionless magnetic parameters, indicated how large the magnetic effects would have to be for this experiment. The limits of the increase in magnetic effects were imposed by the equipment available. It was not obvious at the outset which dimensionless magnetic parameter was the appropriate one to use for correlating earlier experimental results. Previous analytical and experimental work had employed both the Hartmann number and the interaction parameter in the interpretation of results. An examination of the relevant parameters in terms of their physical interpretations indicated which parameters were important for the present case.

The original dimensionless parameter involving the magnetic field was introduced by Hartmann (1937) and is called the Hartmann number,

$$M = B_0 l(\sigma/\eta)^{\frac{1}{2}}, \tag{1}$$

where B_0 is the applied field, l is an appropriate characteristic length, σ is the electrical conductivity and η is the viscosity. The Hartmann number can be shown to be square root of the ratio of magnetic forces to viscous forces, and thus it would be expected to be the important parameter in cases where inertial effects are small. This is the case, for example, in fully developed, laminar, transverse field, MHD channel flow.

If, on the other hand, the viscous forces are not important, the interaction parameter,

$$I = \sigma B_0^2 l/\rho\overline{u}, \tag{2}$$

where $\overline{u}$ is a characteristic velocity and ρ is the fluid density, is the important parameter since it represents the ratio of magnetic to inertial forces. The interaction parameter is the important parameter when dealing with inviscid flow or free turbulence, for example.

A third parameter, less commonly found in MHD analyses, is

$$M/R = B_0(\sigma\eta)^{\frac{1}{2}}/\rho\overline{u}, \tag{3}$$

where R is the ordinary Reynolds number. This parameter represents a mixture of forces and involves viscous, magnetic, and inertial forces. M/R can be thought of as the square root of the product of the viscous and magnetic forces divided by the inertial forces. Thus, the magnitudes of both the viscous and magnetic forces are compared to the magnitude of the inertial forces. If either is small compared to the inertial forces, the parameter is small. This parameter is appropriate for flows in which viscous, magnetic and inertial forces all play important roles.

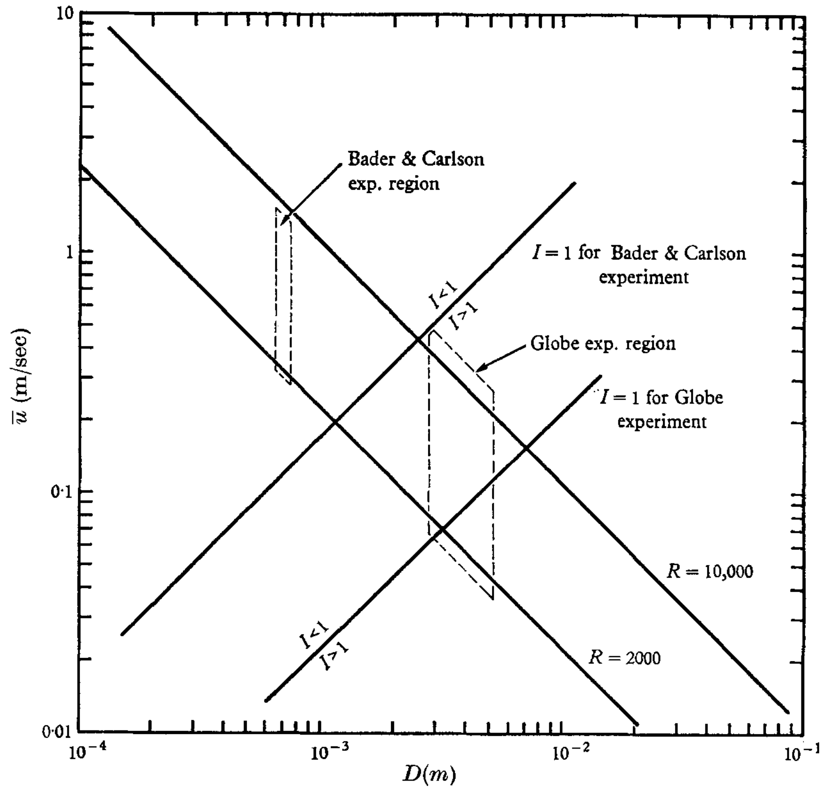

FIGURE 1. Operating characteristics of previous experiments.

For the case of turbulent pipe flow, it was not obvious at the beginning of this study which parameter should be used. Previous work emphasized the importance of turbulent damping in the core of the pipe where viscous effects are small, and thus the interaction parameter was employed. However, it was equally likely that the major effect of the field would be felt on the generation of new turbulence near the walls. In this case M/R, which has both viscous and magnetic dependence, would be more appropriate.

The design of the experiment was eventually based upon the magnetic interaction parameter because it was felt that damping of the turbulence directly by the magnetic field was certain to have an important effect upon the results. In order to examine Globe's and Bader & Carlson's experiments in terms of the interaction parameter and to determine what magnitude of the interaction parameter was necessary for a new experiment, the diagram in figure 1 was found to be helpful. The tube size D and the mean velocity $\bar{u}$ are taken as variables. The interaction parameter is the third variable and is set equal to 1 for the maximum field strengths used in the two previous experiments. The Reynolds number range of 2000 to 10,000 covers the most important experimental data.

The Bader & Carlson experiment, as figure 1 indicates, had an interaction parameter always much less than one, and, as pointed out above, indicated no significant changes in the flow. Globe's experiment had a maximum interaction parameter of about two, and he reported a definite influence on the flow. This dependence of magnetic effects on the interaction parameter indicated that significantly new results could be obtained if the interaction parameter were increased to ten or more, and this became the major objective in the design of the experiment.

A plot similar to figure 1 showing the operating regions for a mercury system, with the interaction parameter based on the maximum field available (1·8 w/m²), was used to choose the most suitable tube size, which was the only variable left once the working fluid, magnetic field strength, and Reynolds number range were specified. Without any further restriction on the system, there would have been no limit on the tube size for the experiment. The larger the diameter of the tube, the greater is the interaction parameter. However, since the laminar pressure gradient varies as D^{-3} at constant Reynolds number, the lower limit on the pressure gradient which could be measured accurately imposed an additional restriction on the tube size for the experiment. With available measuring equipment and reasonable pressure tap spacing along the tube, the minimum pressure gradient which could be measured within $\pm 5\,\%$ was 2×10^{-4} m Hg/m. The turbulent pressure gradient would always be greater than the laminar and thus imposed less of a restriction.

Figure 2 indicates the results of the present design study. The interaction parameter $(\sigma B_0^2 D/\rho\bar{u})$ with values of one and ten is indicated for a magnetic field strength of 1·8 w/m².

Reynolds numbers below approximately 2000 should yield results which are independent of magnetic field strength and thus serve as a check on the data. For this reason, these low Reynolds number pressure gradient data were important enough that considerable accuracy was necessary in the Reynolds number range 1000 to 2000. The limiting pressure gradient line shown is given by

$$dH/dx = 32\nu\bar{u}/gD^2 = 2 \times 10^{-4}\,\mathrm{m\,Hg/m}, \tag{4}$$

where dH/dx is the axial pressure gradient, g is the gravitational constant and ν is the kinematic viscosity. A tube diameter of 6·35 mm was chosen from the range of tube sizes which fitted all the stated requirements, and this choice is indicated in figure 2.

Because it was desired to make accurate transition measurements for the flow, a hot-wire anemometer was used to detect the onset of turbulence. The hot-wire used was designed specifically for use in mercury (Sajben 1965). Due to its design, the sensitivity of the hot-wire decreased as the velocity increased, and there was, therefore, a maximum velocity beyond which the hot-wire could not be used. For the chosen design, turbulence could be accurately detected for Reynolds numbers as high as 10,000, which was well suited to the experiments.

Finally, it should be noted that the magnetic Reynolds number, which indicates the magnitude of induced magnetic fields relative to applied fields, was always less than 0·01 in these experiments, so that the applied fields could safely be considered to be undisturbed.

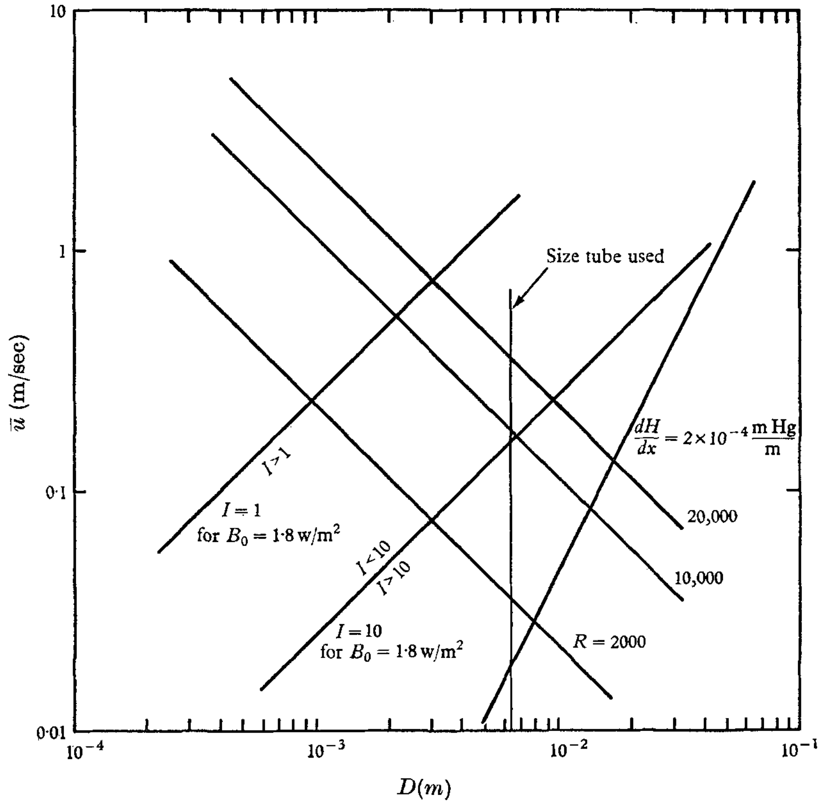

FIGURE 2. Operating characteristics for mercury system using $B_0 = 1{\cdot}8$ w/m and minimum $dH/dx = 2\times10^{-4}$ m Hg/m.

3. Equipment design

The system used (figure 3) was essentially a gravity flow type with a supply tank feeding mercury to a 140 cm long test section. The flow then emptied into a receiving reservoir. A peristaltic pump returned the mercury to the supply tank.

The upper reservoir was equipped with an overflow which produced a constant head in the tank. To measure flow rate, the pump was stopped and, once the overflow had stopped, the rate of decrease of the mercury head was measured by means of a pressure transducer. This method provided a simple, accurate means of determining the flow rate through the test section. The scatter in the flow rate measurements was less than 2 % for all flow conditions.

The test section consisted of three 47 cm long glass tubes placed end to end. This tubing was specially ground to have a precision bore of 6·35 mm ± 0·005 mm. The sections were joined in such a way that there was no misalignment in the bores of the three tubes. The downstream section of tubing contained five pressure taps spaced 10 cm apart, which were used to measure the pressure gradient. The overall length of the test section was set by the length of the uniform magnetic field available in the magnet used, the experiment being designed so that the entire test section was in the uniform portion of the field. In this region, measurements showed that the field varied by no more than ± 2 %.

The magnet used was a ten coil solenoidal magnet with an inside diameter of 6 in. The overall length of the coils was 62 in. and the uniform portion of the field was approximately 56 in.

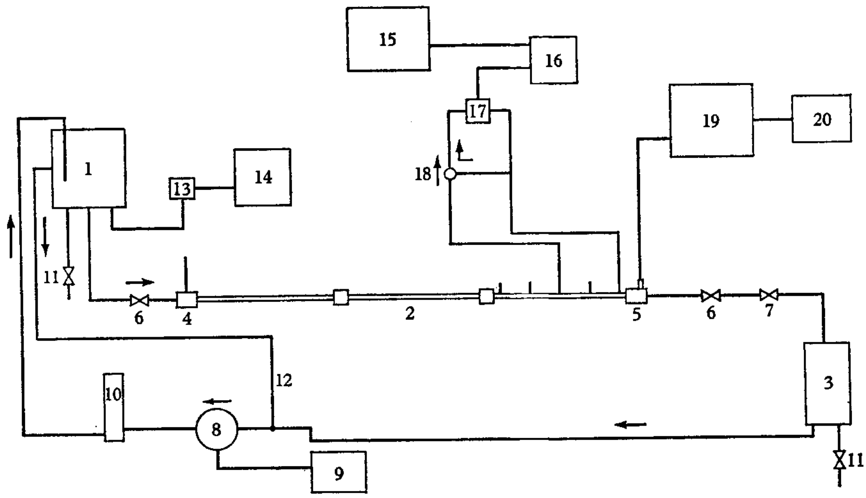

FIGURE 3. Diagram of system. 1, upper supply tank; 2, three-part test section; 3, lower tank; 4, entrance fitting; 5, hot-wire; 6, shut-off valves; 7, caliper valve; 8, pump; 9, pump control; 10, accumulator; 11, drain valves; 12, overflow line; 13, pressure transducer (P.T.); 14, Sanborn trans. converter; 15, $L+N$ $K3$ potentiometer; 16, trans. converter; 17, differential P.T.; 18, three-way valve; 19, hot-wire bridge; 20, strip chart recorder.

The hot-wire which was used to detect turbulence was positioned immediately downstream of the pressure taps and was also in the uniform field. Figure 4 indicates how the hot-wire probe was positioned in the system. The bore of the hot-wire mount matched the test section bore and was carefully aligned with it. The hot-wire itself extended across the diameter of the bore. This design was sensitive to transition, but the design precluded any attempt to measure local turbulence.

It should be pointed out that, because of the proximity of the entrance of the test section to the strong fringing field at the entrance to the magnet solenoid, no attempt was made to achieve a quiet entrance flow. Quite to the contrary, the entrance fitting, indicated in figure 3, was designed to generate a disturbed flow at the entrance to the test section. Thus, it was expected that the transition data would correspond to the lower limit of transition Reynolds numbers.

The differential pressures were measured using a Sanborn Co. model 268B differential pressure transducer. This transducer allowed the minimum pressure drop encountered to be measured with a $\pm 5\%$ accuracy. For Reynolds numbers above 1500, the typical scatter in the data for pressure drop was found to be less than 2%.

Because of the extremely small pressures being measured, great care had to be taken in setting up the pressure measuring system so as to avoid thermal effects caused by the hot magnet coils. However, as the system was set up, these temperature effects were so small that they could not be detected in the measurements. Careful measurements also verified that the magnetic field did not disturb the pressure measurements to any measurable degree.

It was necessary that the temperature be known in order to ensure that correct values for the physical properties of mercury were used. While the variation in the density of mercury is quite small, approximately 0·02 %/°C, the viscosity has a variation of 0·3 %/°C.

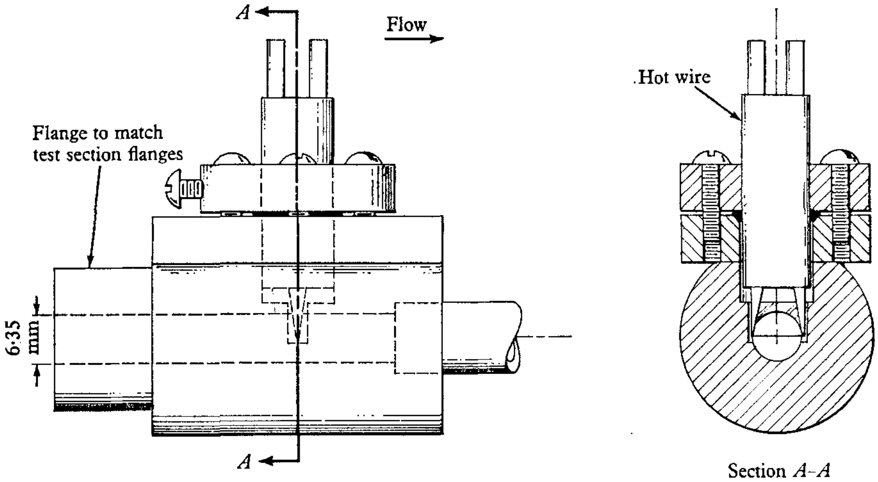

FIGURE 4. Hot wire mounting block.

4. Experimental procedure

The experimental data, a discussion of which follows, were obtained with the system set up as described in the proceeding section. The differential pressure data were obtained, in all but one case, using the downstream pressure tap and another 20 cm upstream. This arrangement was used to provide the maximum flow development length possible. The importance of a large entry length will shortly be made clear.

As data were being taken, the temperature of the mercury was measured periodically. It was found that the temperature limits for the whole experiment were 25 °C ± 3 °C. Thus, rather than having been adjusted continuously, the value of the viscosity was taken as that corresponding to a temperature of 25 °C. This approximation contributed less than ± 1 % scatter to all the data obtained.

The magnetic field, flow rate, and pressure gradient were measured for a large number of flow conditions, and the intermittency factor γ was measured when the flow was in the transition region. The values of γ were obtained from strip chart recordings of the hot-wire output. When these measurements were being made, the Reynolds number was varied in increments of 100 to 200, allowing several values of γ to be obtained through each transition region.

The data were obtained in a series of runs. Each run consisted of a set of pressure drop-flow rate data with the magnetic field held constant. The hot-wire was operated only when the flow was in the transition region in order to avoid any unnecessary heating of the mercury. Data were obtained by both continuously increasing and continuously decreasing the flow rate, and no hysterisis was found in the data. While flow data were being taken, the peristaltic pump was not operated, but the large diameter reservoir assured an almost constant head over the period of measurement.

For certain flow rates, particularly for fully turbulent flow and flow in the transition region, the output of the differential pressure transducer was unsteady. For the fully turbulent flow, the output was close to a steady oscillation, and thus an average reading was recorded; the oscillation was small compared to the magnitude of the signal. In the transition region no accurate pressure reading could be obtained. It was possible to read a high and low limit on the output, however. The average of these two extremes was used when data in the transition region were reported. These points are intended to be only representative.

5. Discussion of results and conclusions

Laminar flow: entry length effects

In order to ensure the reliability of all the data, it was necessary to refer to the data obtained for the case of no magnetic field since these data tested the calibration of the experiment. Figure 5 is the plot of the friction factor–Reynolds number data with no magnetic field. It is evident that the system yielded results in

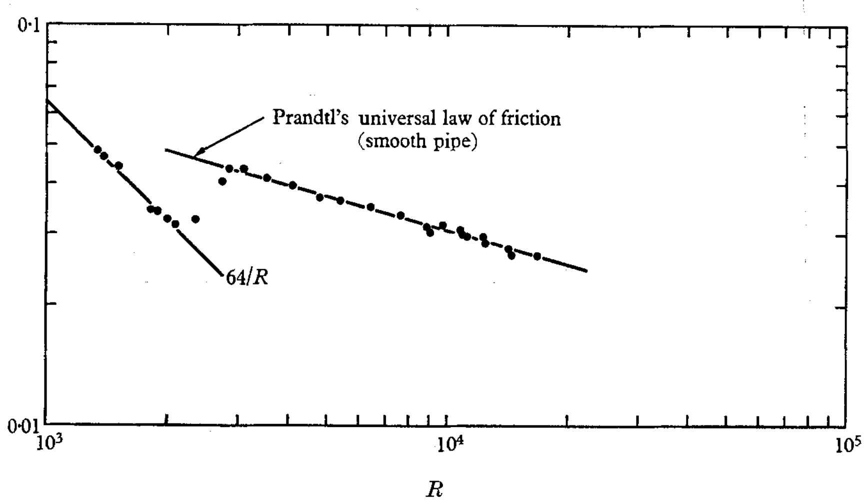

FIGURE 5. Friction factor data for $B_0 = 0$.

excellent agreement with established information over the entire Reynolds number range of the experiment. It should also be added that the results of the hot-wire intermittency factor measurements yielded a transition Reynolds number of 2250. The exact definition of transition Reynolds number used here will be given below.

Figure 6 is a plot of the friction factor–Reynolds number results for several magnetic field strengths. The data in this plot are for laminar flow only. All the data were obtained with a flow development length to diameter ratio x/D of

180. The flow development length is defined as the distance between the test section entrance and the first pressure tap that was used.

As has been noted previously, the magnetic field should have no effect on the fully developed laminar flow. The absence of magnetic effects for Reynolds

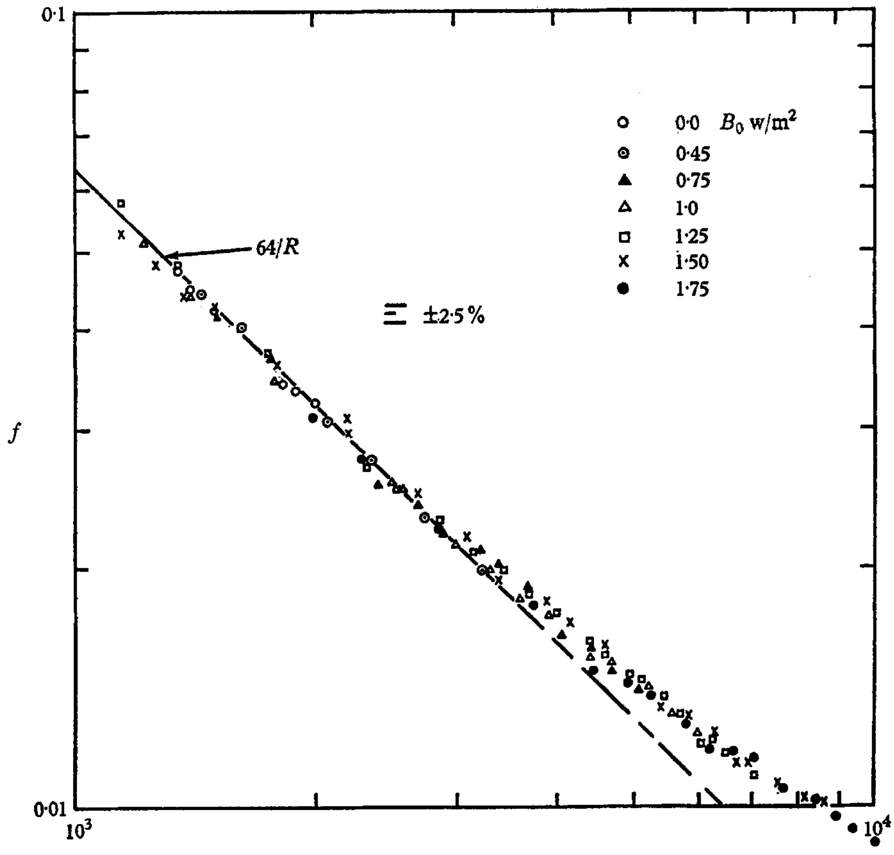

FIGURE 6. Laminar flow data.

numbers less than 2800 may be taken as evidence of insignificant transverse magnetic field components. The deviation of the data from the theoretical results, evident in figure 6, is due to entry effects which become important as the Reynolds number increases. The non-MHD flow analysis of Goldstein (1938) shows that the laminar flow is fully developed as long as the ratio $(x/D)/R$ is greater than 0·06. Since x/D was constant in this experiment and since increasing the magnetic field allowed laminar flow to be maintained at higher Reynolds numbers, a point was eventually reached at which the ratio $(x/D)/R$ became equal to 0·06. For Reynolds numbers greater than this value, the laminar flow was not fully developed. For an x/D of 180, the corresponding R is 3000 according to this criterion. The data indicate that deviation from the fully developed pressure drop started at a Reynolds number of about 2800.

Some interesting MHD effects can be found from a close inspection of the data of figure 7. The tendency of the magnetic field to suppress radial velocities in the entrance region should cause the flow to develop more slowly. Slower development of the velocity profile, in turn, should increase the skin friction at any point

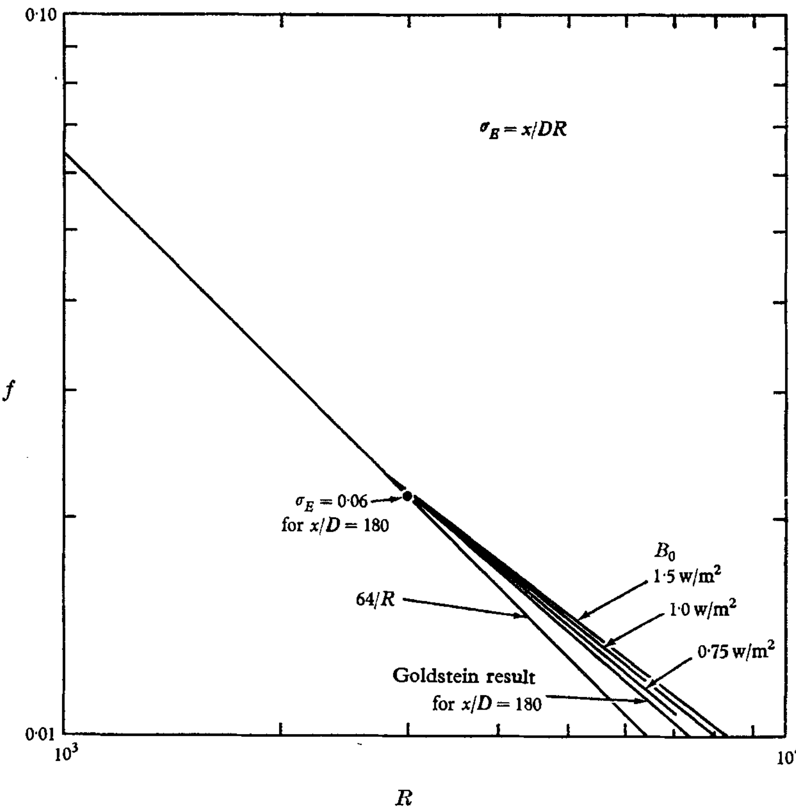

FIGURE 7. Comparison of laminar data to Goldstein analysis for $x/D = 180$.

in the tube as the magnetic field increases at a given Reynolds number. This trend is certainly apparent in the laminar data. In fact, as the magnetic field approaches zero, the data approaches the theoretical non-MHD result given by Goldstein. This effect should be somewhat counteracted by the fact that the inlet flow was generally turbulent, which would also tend to increase the skin friction at any point in the tube at a given Reynolds number, and the fact that increasing the magnetic field would tend to reduce this effect by suppressing the inlet turbulence. However, the analysis of Deissler (1963) may be used to show that the turbulence could not have persisted for more than 15 diameters even for the smallest magnetic field employed, and this distance decreased rapidly with increasing magnetic field. Consequently, it is not surprising that the data seem to reflect the effect of inlet turbulence only by a slight downward shift in the Reynolds number at which deviation from the fully developed friction factor begins.

It should be noted that the fact that the flow is not quite fully developed at the higher values of magnetic field will tend to cause an apparent increase in the ability of the field to prevent transition. However, as the flow was always sufficiently well developed, this effect should have been slight.

Transition region

For flow in the transition region, the hot-wire was used to determine the intermittency factor γ of the flow. The intermittency factor was obtained from recordings of the hot-wire output at closely spaced Reynolds numbers within the

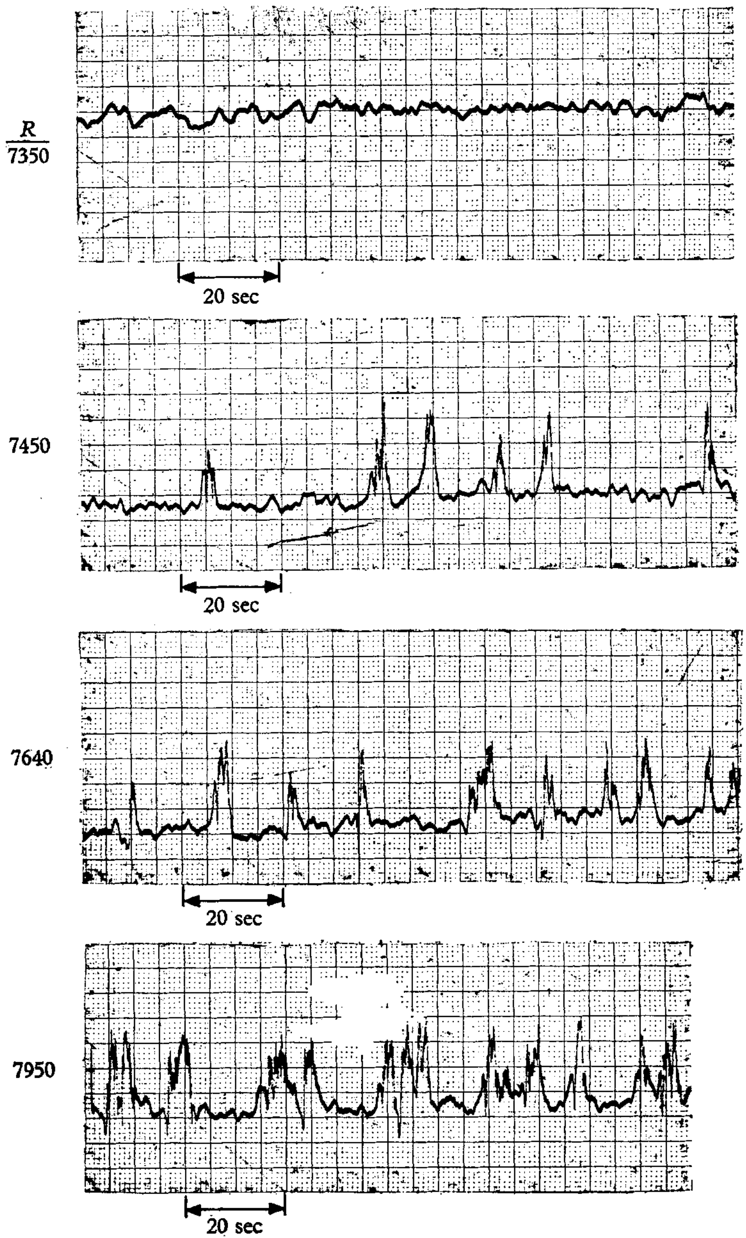

FIGURE 8. Hot-wire output for B_0 = 12,500 gauss.

transition region, and it was defined as the percentage of the time that the flow was turbulent. Figure 8 is a typical set of hot-wire recordings. It should be made clear that the flow at $R = 7350$ is laminar, albeit somewhat unsteady. The frequency of the unsteadiness was of the order of 0·5 c/s and was much lower than

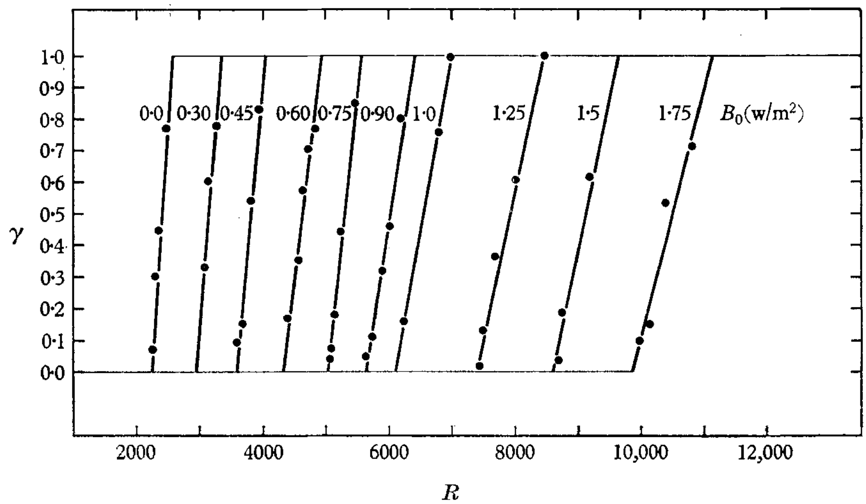

FIGURE 9. Intermittency factor data.

the typical frequencies of the turbulent slugs in the transition region. This low frequency unsteadiness was due to the character of the flow outside of the magnetic field. Even though laminar flow existed within the magnetic field, the flow rapidly became turbulent downstream of the test section and gave rise to small-amplitude low frequency unsteady flow within the test section.

Figure 9 presents the intermittency factor data as a function of the Reynolds number for all the magnetic fields used. The actual curve that the data followed could be reasonably approximated by a straight line. To determine the value of the Reynolds number at transition R_t, the intercept of the γ line with the $\gamma = 0$ axis was used. These values of R_t are plotted as a function of the interaction parameter in figure 10. The interaction parameter is that which corresponds to R_t and is designated I_t. The transition Reynolds number is conveniently represented by the ratio R_t/R_0, where R_0 is the non-MHD transition Reynolds number value of 2250. The experimental results can be accurately summarized by the relationship

$$R_t/R_0 = 1 + 0{\cdot}40 I_t. \tag{5}$$

The maximum Reynolds number at which laminar flow was maintained was 10,350, the corresponding interaction parameter being 9·0.

Also plotted in figure 10 are the data from Globe's experiment and the results of Stuart's analysis for stability between parallel plates. As was mentioned earlier, it was not expected that Stuart's theory would yield results applicable to the present experiment because of the geometry of the case he treated. It can be seen that the parallel plate analysis shows much greater stabilizing effects than are actually observed. On the other hand, Globe's results are in essential agreement with the present results. It was expected that Globe's results would show more scatter than the present results because he relied upon the onset of unsteadiness of pressure readings to detect transition. Since his pressure measurement technique was inertial, the magnetic field could have had an important influence on his observations. Furthermore, transition outside the magnetic field would have contributed to any unsteadiness he observed.

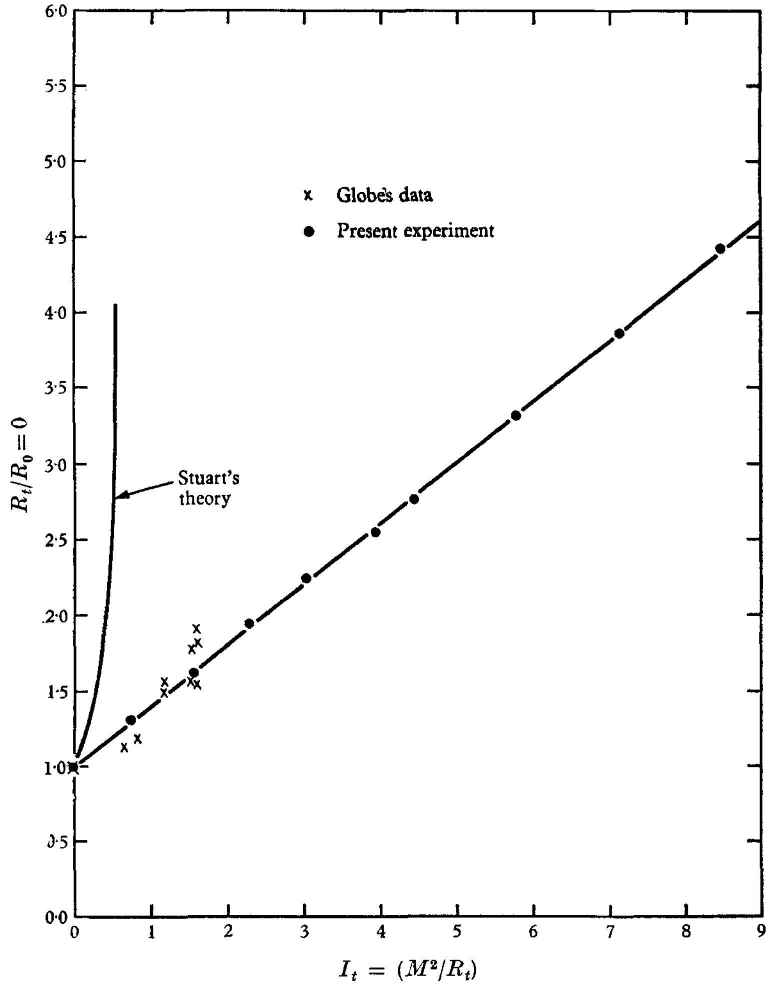

FIGURE 10. Dependence of transition Reynolds number ratio on I_t, $R_0 = 2250$.

Fully developed turbulent flow

Since no complete analysis was available for the prediction of the behaviour of the fully developed turbulent flow, the raw data were arranged according to several schemes in an attempt to gain some understanding of the important characteristics of such flows.

Figure 11 presents the relationship between friction factor and Reynolds number for constant values of the magnetic field strength. This format presents the data in the form in which it was obtained, and the lines of constant field may be thought of as lines of constant Hartmann number. Figures 12 and 13 present the relationship between friction factor and Reynolds number for constant values of the interaction parameter and M/R, respectively.

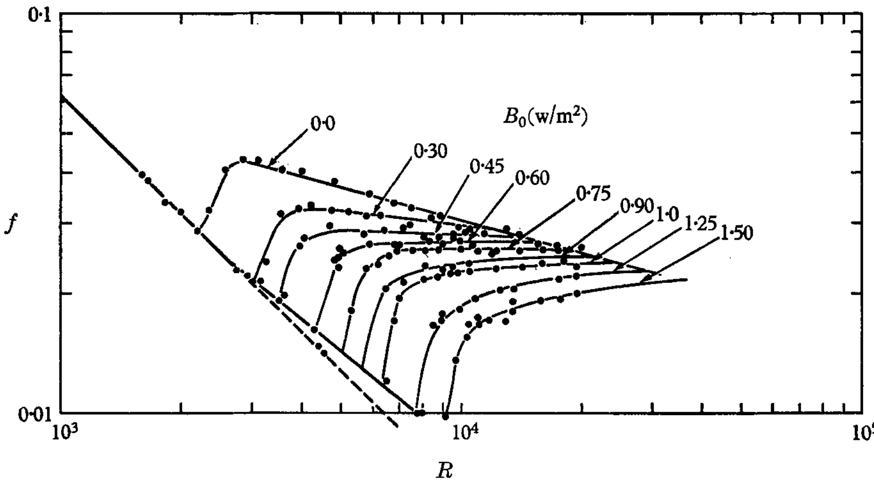

FIGURE 11. Turbulent friction factor data for constant B_0.

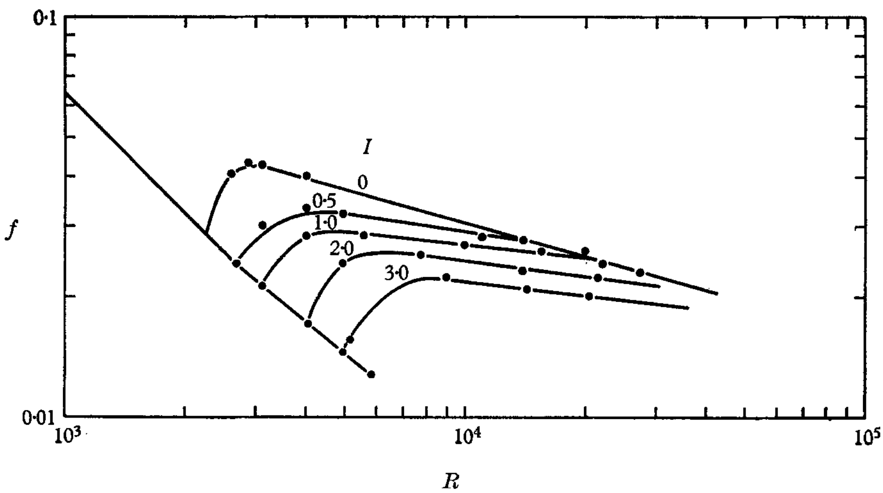

FIGURE 12. Turbulent friction factor data for constant I.

Figures 11 and 12 reveal that whatever the constant value of M or I, R can always be made sufficiently large so that the magnetic field no longer has an effect upon f. An important conclusion to be drawn from these observations is that the magnitude of induced MHD forces relative to inertial forces does not control the structure of the fully developed turbulence. Consequently, the acknowledged tendency of the magnetic field to dissipate turbulence cannot be regarded as the most important characteristic of such flows as far as wall friction is concerned.

In contrast to this result, figure 13 shows that lines of constant M/R tend to fan out between the usual turbulent line and the theoretical laminar line. This indicates that the friction factor is largely affected by the combined action of viscous

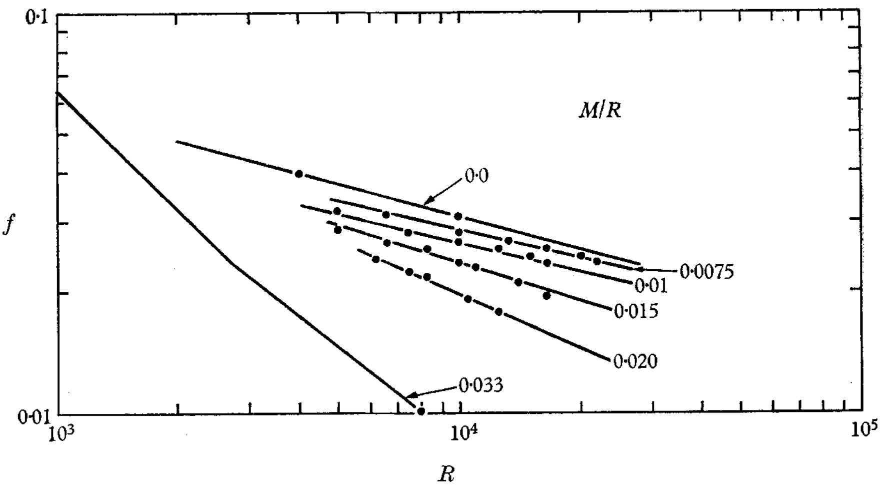

FIGURE 13. Turbulent friction factor data for constant M/R.

and magnetic effects and suggests strongly that the induced MHD forces tend to stabilize the viscous layer near the wall and thereby reduce the rate of generation of turbulence. This conclusion is supported by the transition Reynolds number results which, when $I_t \gg 1$, are given by the expression

$$(M/R)_t = 0{\cdot}033. \tag{6}$$

Although the nature of these experiments was such that only the overall behaviour of the fully developed turbulent flow was revealed, the results show that an analysis is required which accounts for both the generation and the suppression of turbulence. Until such an analysis becomes available, experimental data must be relied upon.

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REFERENCES

BADER, M. & CARLSON, W. C. A. 1958 *NACA TN* 4274.

DEISSLER, R. G. 1963 *Physics Fluids*, **6**, 1250.

GLOBE, S. 1961 *J. Heat Transfer*, **83**, 445.

GOLDSTEIN, S. 1938 *Modern Developments in Fluid Mechanics*, vol. I, Oxford: Clarendon Press.

HAINS, F. D. 1965 *Physics Fluids*, **8**, 2014.

HARRIS, L. P. 1960 *Hydrodynamic Channel Flows*. Tech. Press of M.I.T. and J. Wiley and Sons.

HARTMANN, J. 1937 *Danske Videnskab. Selskab. Mat.-fys. Medd.* **15**, 6.

HARTMANN, J. & LAZARUS, F. 1937 *Danske Videnskab. Selskab. Mat.-fys. Medd.* **15**, 7.
HUNT, J. C. R. 1966 *Proc. Roy. Soc.* A **293**, 342.
LOCK, R. C. 1955 *Proc. Roy. Soc.* A **233**, 105.
MICHAEL, D. H. 1953 *Proc. Camb. Phil. Soc.* **49**, 166.
MURGATROYD, W. 1953 *Phil. Mag.* **44**, 1348.
ROSSOW, V. J. 1958 *NASA TR R*-37.
SAJBEN, M. 1965 *Rev. Scient. Instrum.* **36**, 945.
STUART, J. T. 1954 *Proc. Roy. Soc.* A **221**, 189.